

Forbidden and invisible Z boson decays in a covariant θ -exact noncommutative standard model

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Triple neutral gauge boson and direct photon-neutrino interactions, being absent in ordinary field theory, arise quite naturally in noncommutative gauge field theories. Using non-perturbative methods and a Seiberg-Witten map based covariant approach to noncommutative gauge theory, we found θ -exact expressions for the interactions, thereby eliminating previous restrictions to low-energy phenomena. In particular we obtain for the first time covariant θ -exact triple neutral gauge boson interactions within the noncommutative Standard Model gauge sector. Finally we discuss implications for $Z \rightarrow \gamma\gamma$ and $Z \rightarrow \bar{\nu}\nu$ decays, and show that our results behave reasonably throughout all interaction energy scales.

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I. INTRODUCTION

After the string theory indicated that noncommutative (NC) gauge field theory (GFT) could be one of its low-energy effective theories [1], the studies on noncommutative particle phenomenology started its development [2, 3]. Aim was to find possible experimental signatures and predict/estimate bounds on spacetime noncommutativity from collider physics experimental data, in particular from the Standard Model (SM) forbidden $Z \rightarrow \gamma\gamma$ decay mode and invisible part of $Z \rightarrow \bar{\nu}\nu$ decays.

Significant progress has been obtained in the Seiberg-Witten (SW) maps and enveloping algebra based models where one could deform commutative gauge theories with arbitrary gauge group and representation [4–11]. The constraints on the $U_*(1)$ charges (erratically stated in some earlier literatures as “no-go theorem” [12]) are also rescinded in this approach, see details in section two of [11]. The noncommutative extensions of particle physics models like the covariant noncommutative SM (NCSM) and the NC GUT models [10, 11, 13–17] were constructed. These allow a minimal (without new particle content) deformation with the sacrifice that interactions include infinitely many terms defined through recursion over (and in practice cut-off at certain order of) the NC parameter $\theta^{\mu\nu}$. There were also other studies with NC modifications of particle physics [18–22].

Studies of divergences have been performed since the early stage of the NC QFT development [23]. Later well behaving one-loop quantum corrections to noncommutative scalar ϕ^4 theories [24–26] and the NC QED [27] have been found. Also the SW expanded NCSM [10, 13, 15, 17] at first order in θ , albeit breaking Lorentz symmetry is anomaly free [28], and has well-behaved one-loop quantum corrections [18, 29–35].

At θ -order there are two important interactions that are suppressed and/or forbidden in the SM, the triple neutral gauge boson [13, 15, 17], and the tree level cou-

pling of neutrinos with photons [36, 37], respectively. Here an expansion and cut-off in powers of the NC parameters $\theta^{\mu\nu}$ corresponds to an expansion in momenta and restrict the range of validity to energies well below the NC scale Λ_{NC} . Usually, this is no problem for experimental predictions because the lower bound on the NC parameters $\theta^{\mu\nu} = c^{\mu\nu}/\Lambda_{\text{NC}}^2$ (the coefficients $c^{\mu\nu}$ running between zero and one) runs higher than typical momenta involved in a particular process. However, there are exotic processes in the early universe as well as those involving ultra high energy cosmic rays [38–41] in which the typical energy involved is higher than the current experimental bound on the NC scale Λ_{NC} . Thus, the previous θ -cut-off approximate results are inapplicable. To cure the cut-off approximation, we are using θ -exact expressions, inspired by exact formulas for the SW map [8, 42–45], and expand in powers of gauge fields, as we did in [40]. In θ -exact models we have studied the UV/IR mixing [46, 47], the neutrino propagation [48] and also some NC photon-neutrino phenomenology [38–41], respectively. Due to the presence of the UV/IR mixing the θ -exact model is not perturbatively renormalizable, thus the relations of quantum corrections to the observations [49] are not entirely clear.

To study triple neutral gauge boson couplings, in this letter we construct the θ -exact pure gauge sector action of the $SU(2) \times U(1)$ gauge group, while for the NC Z-boson-neutrino and photon-neutrino interactions we are using actions from [11]. We compute only tree level processes and therefore treat the NC model as an effective theory only. The decay of Z boson into two photons has been used to predict possible experimental signatures of the NCSM [13, 15, 17, 22]. Since by Bose-symmetry and rotational invariance arguments [50] any vector particle cannot decay into two massless vector particles, this forbidden decay has very little background from the standard model. Fixing θ spontaneously breaks C, P, and/or CP discrete symmetries [16]. See also general discussions on the C,P,T, and CP properties of the noncommutative

interactions in [51], and in the case of our model a discussions given in [52, 53]. A breaking of C symmetry occurs in $Z \rightarrow \gamma\gamma$ process. One common approximation in those existing works is that only the vertices linear in terms of the NC parameter θ were used. In this work we extend the NCSM gauge sector actions for first time to all orders of θ . Next we discuss the decay widths $\Gamma(Z \rightarrow \gamma\gamma)$ and $\Gamma(Z \rightarrow \nu\nu)$ as functions of the NC scale Λ_{NC} for space/light-like noncommutativity which are allowed by unitarity condition [54, 55].

II. NCSM GAUGE SECTOR IN A θ -EXACT MODEL

As usual we consider the star product formalism for quantum field theory on the deformed Moyal space with a constant noncommutative parameter θ . We start with a θ exact expansion of the noncommutative gauge field \hat{V}_μ in the terms of component fields,

$$\begin{aligned} \hat{V}_\mu = & V_\mu^a T^a - \frac{1}{8} \theta^{ij} \left[\left\{ V_i^a \star_2 (\partial_j V_\mu + F_{j\mu})^b \right\} \left\{ T^a, T^b \right\} \right. \\ & \left. + \left\{ V_i^a \star_{2'} (\partial_j V_\mu + F_{j\mu})^b \right\} i f^{abc} T^c \right]. \end{aligned} \quad (1)$$

The generalized star products \star_2 and $\star_{2'}$ with the properties: $f \star_2 g = g \star_2 f$, $f \star_{2'} g = -g \star_{2'} f$, $[f \star_2 g] = i\theta^{ij} \partial_i f \star_2 \partial_j g$, $\{f \star_2 g\} - \{f, g\} = \theta^{ij} \partial_i f \star_{2'} \partial_j g$, are defined in [11, 42]. Besides the \star -commutator term, starting at first order of θ , one gets a \star -anticommutator term starting at second order in θ . The pure gauge action expanded in terms of the component fields reads:

$$\begin{aligned} S_g = & \frac{-1}{2} \int \sum_{a,b} \text{tr}(T_a T_b) F_{\mu\nu}^a F^{b\mu\nu} - \sum_{a,b,c} \left\{ \text{tr} T_a \{T_b, T_c\} \right. \\ & \cdot F^{a\mu\nu} \left[\theta^{ij} \partial_\mu \left(V_i^b \star_2 (\partial_j V_\nu^c + F_{j\nu}^c) \right) + i[V_\mu^b \star V_\nu^c] \right] \\ & + \text{tr} T_a [T_b, T_c] F^{a\mu\nu} \left[i \left(\{V_\mu^b \star V_\nu^c\} - \{V_\mu^b, V_\nu^c\} \right) \right. \\ & \left. \left. - \theta^{ij} V_i^b \star_{2'} (\partial_j V_\nu^c + F_{j\nu}^c) \right] \right\}. \end{aligned} \quad (2)$$

The two traces $\text{tr} T_a [T_b, T_c]$ and $\text{tr} T_a \{T_b, T_c\}$ are both well known in the representation theory of Lie algebras. The first one is proportional to the structure constant f_{abc} , with the quadratic Casimir as its coefficient, $\text{tr} T_a [T_b, T_c] = i \text{tr} T_a f_{abc} T_d = i A f_{abc}$. Thus the different gauge sectors do not mix in this part of the action. The second trace is slightly more complicated as it is connected with a detailed representation of the gauge group

(we denote it as B_{abc}):

$$\begin{aligned} S_g = & \int \sum_a A F_{\mu\nu}^a F^{a\mu\nu} \\ & - \sum_{a,b,c} B_{abc} F^{a\mu\nu} \left[\theta^{ij} \partial_\mu \left(V_i^b \star_2 (\partial_j V_\nu^c + F_{j\nu}^c) \right) \right. \\ & \left. + i[V_\mu^b \star V_\nu^c] \right] + \dots = S_{\text{gauge}} + \dots \end{aligned} \quad (3)$$

Using the generalized star product \star_2 , we can rewrite the star commutator as $i[V_\mu^b \star V_\nu^c] = -\theta^{ij} \partial_i V_\mu^b \star_2 \partial_j V_\nu^c$. After a series of partial integrations and by noting that B_{abc} is totally symmetric under the permutations of a, b and c , one arrives at

$$\begin{aligned} S_{\text{gauge}} = & \int A F_{\mu\nu}^a F^{a\mu\nu} \\ & - B_{abc} \theta^{ij} F^{a\mu\nu} \left(\frac{1}{4} F_{ij}^b \star_2 F_{\mu\nu}^c - F_{\mu i}^b \star_2 F_{\nu j}^c \right). \end{aligned} \quad (4)$$

The cubic Casimir B_{abc} is in general group and representation dependent. It reflects a few common properties:

- B_{abc} takes on the opposite sign for a representation and its complex conjugate;
- B_{abc} vanishes for the adjoint representation of any Lie group;
- B_{abc} vanishes for any simple Lie group except $\text{SU}(N \geq 3)$.

For this reasons, $\text{SO}(10)$ and E_6 GUT models have no additional noncommutative triple gauge boson couplings which are forbidden in the standard model (this attribute was qualified as the “uniqueness” of the noncommutative GUT model in [16]). Furthermore when the trace in the standard model is computed using generators descended from non- $\text{SU}(N \geq 3)$ simple gauge groups, for example from $\text{SO}(10)$ or E_6 GUTs, there is no such coupling either. The standard model and $\text{SU}(5)$ GUT were studied before [13, 14, 16, 17] as both could accommodate the neutral boson mixing couplings $\gamma\gamma\gamma$, $Z\gamma\gamma$, $ZZ\gamma$, γGG , ZGG , and ZZZ . Since in $\text{SU}(5)$ GUT all heavy gauge bosons are charged, the standard model neutral boson coupling has covered all the cases. To introduce possible extra neutral gauge bosons, one may consider models with left-right symmetry like the Pati-Salam $\text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$ [56] or trinification $\text{SU}(3)^3 \times \text{Z}_3$ [57]. In the Pati-Salam model, the vanishing of B_{abc} for $\text{SU}(2)$ forces all mixing to arise from the $\text{SU}(4)$ sector, that is, YYY , YGG (and GGG) couplings only, thus no mixing will include heavy gauge bosons. The $\text{SU}(4)$ sector has either $\mathbf{4}$ or $\bar{\mathbf{4}}$ representation. Therefore, up to normalization the following terms

$$\begin{aligned} \text{tr} Y \{Y, Y\} &= -\frac{8}{9}, \quad \text{tr} Y \{G^a, G^b\} = \delta^{ab}, \\ \text{tr} G^a \{G^b, G^c\} &= d_{(3)}^{abc}, \end{aligned} \quad (5)$$

are the only non-vanishing neutral-boson coupling components. The trinification $SU(3)^3 \times Z_3$ seems more promising since the left and right symmetry group are both $SU(3)$. However, in this model, all matter multiplets are of Z_3 symmetric $(3, \bar{3}, 1) \oplus (\bar{3}, 1, 3) \oplus (1, 3, \bar{3})$ type. Thus, when Z_3 is maintained, all mixing couplings cancel between $\mathbf{3}$ and $\bar{\mathbf{3}}$.¹

Returning to the the standard model gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ we find the non-vanishing elements to be

$$\begin{aligned} \text{tr} Y \{T_L^i, T_L^j\} &= \text{tr} T_L^i \{Y, T_L^j\} = \text{tr} T_L^i \{T_L^j, Y\}, \\ \text{tr} Y \{T_c^i, T_c^j\} &= \text{tr} T_c^i \{Y, T_c^j\} = \text{tr} T_c^i \{T_c^j, Y\}, \\ \text{tr} T_c^i \{T_c^j, T_c^k\} &= A_{2c} \cdot d^{ijk}, \quad \text{and} \quad \text{tr} Y^3. \end{aligned} \quad (6)$$

As the corresponding interaction terms for the standard model matter multiplets, $SU(2)$ and $SU(3)$, both sit in the fundamental representation, the hypercharge generator can be set as $Q_Y \cdot I$ for each generation. Introducing weight $\alpha(\rho)$ for each representation ρ of the matter multiplets [13, 15, 29], one reaches θ -exact result

$$\begin{aligned} S_{\text{gauge}}^\theta &= g_Y^3 \kappa_1 \int \theta^{\rho\sigma} f^{\mu\nu} \left(\frac{1}{4} f_{\rho\sigma} \star_2 f_{\mu\nu} - f_{\mu\rho} \star_2 f_{\nu\sigma} \right) \\ &+ g_Y g_L^2 \kappa_2 \int \theta^{\rho\sigma} \left[f^{\mu\nu} \sum_{a=1}^3 \left(\frac{1}{4} F_{\rho\sigma}^a \star_2 F_{\mu\nu}^a - F_{\mu\rho}^a \star_2 F_{\nu\sigma}^a \right) \right] \\ &+ g_Y g_c^2 \kappa_3 \int \theta^{\rho\sigma} \left[f^{\mu\nu} \sum_{b=1}^8 \left(\frac{1}{4} G_{\rho\sigma}^b \star_2 G_{\mu\nu}^b - G_{\mu\rho}^b \star_2 G_{\nu\sigma}^b \right) \right] \\ &+ c.p., \end{aligned} \quad (7)$$

where couplings κ_i , $i = 1, 2, 3$, and $K_{Z\gamma\gamma}$, are obtained in [13, 15]. Starting from (7), after straightforward reading-out procedure, the resulting θ -exact Feynman rule is:

$$\Pi_{Z\gamma\gamma}^{\mu\nu\nu'}(p; k, k') = -2e \sin 2\theta_W K_{Z\gamma\gamma} F(k, k') V_{Z\gamma\gamma}^{\mu\nu\nu'}(p; k, k'), \quad (8)$$

with $V_{Z\gamma\gamma}^{\mu\nu\nu'}(p; k, k')$, being given in [22], while $F(k, k') = \sin \frac{k\theta k'}{2} / \frac{k\theta k'}{2}$.

III. Z DECAYS

To illustrate certain physical effects of our θ -exact construction, we compute the $Z \rightarrow \gamma\gamma$ and $Z \rightarrow \nu\bar{\nu}$ decay rates in the Z -boson rest frame, which is then

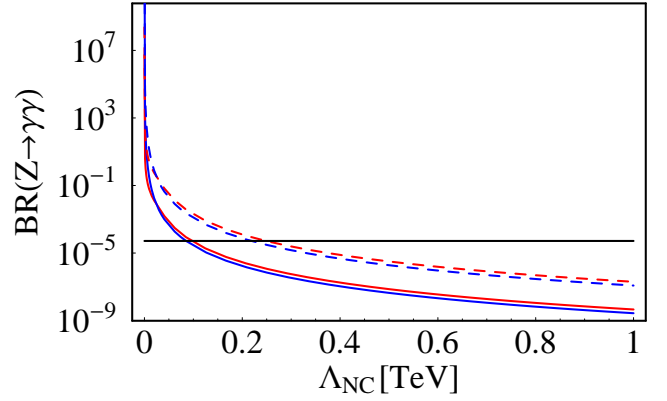


FIG. 1: $BR(Z \rightarrow \gamma\gamma)$ vs. Λ_{NC} . The black horizontal line is the experimental upper limit $BR(Z \rightarrow \gamma\gamma) < 5.2 \cdot 10^{-5}$ [43]. The red and blue curves are explained in the text.

readily to be compared with the precision Z resonance measurements on e^+e^- colliders, where Z is produced almost at rest.

The NCSM $Z \rightarrow \gamma\gamma$ decay rate

The partial $Z \rightarrow \gamma\gamma$ decay width obtained from (8) reads

$$\begin{aligned} \Gamma(Z \rightarrow \gamma\gamma) &= \frac{\alpha \sin^2 2\theta_W K_{Z\gamma\gamma}^2}{3M_Z |\vec{E}_\theta|^2} \left[-2 \left(|\vec{B}_\theta|^2 + 5|\vec{E}_\theta|^2 \right) M_Z^2 \right. \\ &+ |\vec{E}_\theta| \left(|\vec{B}_\theta|^2 + 3|\vec{E}_\theta|^2 \right) M_Z^4 \text{Si} \left(\frac{|\vec{E}_\theta| M_Z^2}{2} \right) \\ &+ 2 \left(|\vec{B}_\theta|^2 + 3|\vec{E}_\theta|^2 \right) M_Z^2 \cos \left(\frac{|\vec{E}_\theta| M_Z^2}{2} \right) \\ &\left. + 8|\vec{E}_\theta| \sin \left(\frac{|\vec{E}_\theta| M_Z^2}{2} \right) \right], \end{aligned} \quad (9)$$

where we have used the following notation $\theta^2 = (\theta^2)_\mu^\mu = \theta_{\mu\nu} \theta^{\nu\mu} = 2 \left(\vec{E}_\theta^2 - \vec{B}_\theta^2 \right)$. In (9) Si is the sine integral function, $\text{Si}(z) = \int_0^z dt \frac{\sin t}{t}$. Expanding Si in power series, one can easily show that $\Gamma(Z \rightarrow \gamma\gamma)$ vanishes at the limit $\theta \rightarrow 0$.

If $|\vec{E}_\theta|$ is set to be zero (space-like noncommutativity, preserving unitarity), the decay width (9) becomes

$$\Gamma(Z \rightarrow \gamma\gamma) = \frac{\alpha}{12} K_{Z\gamma\gamma}^2 M_Z^5 |\vec{B}_\theta|^2 \sin^2 2\theta_W, \quad (10)$$

exactly equal to the decay width in Ref. [22] at $|\vec{E}_\theta| = 0$ and for $a = 1$. This is the expected behavior because the factor $F(k_2, k_3)$ is equal to one in the Z rest frame when $\vec{E}_\theta = 0$, thus showing consistency of the present computations. For light-like noncommutativity (also preserving unitarity [55]) the full NC effect will be still exhibited.

We define the branching ratio $BR(Z \rightarrow \gamma\gamma) = \Gamma(Z \rightarrow \gamma\gamma) / \Gamma_{\text{tot}}$ where $\Gamma_{\text{tot}} = (2.4952 \pm 0.0023) \text{ GeV}$ is the

¹ An analogous conclusion was drawn from the analysis of the NCSM gauge sector at first order in θ [29]. Also note that to exploit the possibility of $Z'\gamma\gamma$ coupling one could probe, for example, the left-right symmetric electroweak model $SU(3)_L \times SU(3)_R \times U(1)_X$ [58].

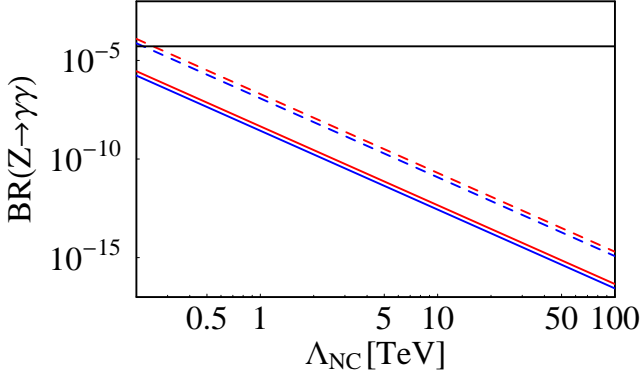


FIG. 2: Same as Fig.1, in the log-log scale.

Z-boson full width [59]. Figure 1 displays $\text{BR}(Z \rightarrow \gamma\gamma)$ based on Eq. (9) for (a) $|\vec{E}_\theta| = 0$ and $|\vec{B}_\theta| = 1/\Lambda_{\text{NC}}^2$ (blue), and for (b) light-like case $|\vec{E}_\theta| = |\vec{B}_\theta| = 1/\sqrt{2}\Lambda_{\text{NC}}^2$ (red). The Z-boson mass is $M_Z = (91.1876 \pm 0.0021)$ GeV [59], the Weinberg angle $\sin^2 \theta_W = 0.23116$, the fine structure constant $\alpha = 1/137.036$. The constant $K_{Z\gamma\gamma}$, defined in [15], is used the same as in Ref. [22], i.e. fixed to $|K_{Z\gamma\gamma}| = 0.05$ and $|K_{Z\gamma\gamma}| = 0.33$ values. In Fig. 1, they are given as solid and dashed lines, respectively. The value $|K_{Z\gamma\gamma}| = 0.05$ was used in Ref. [22] as “the lower central value from the figures and the tables in [15]”. The same figures and tables give 0.33 as the maximal allowed value of $|K_{Z\gamma\gamma}|$. See Fig 2 for the same data displayed in the log-log scale.

The NCSM $Z \rightarrow \nu\bar{\nu}$ decay rate

Since the complete $Z\nu\nu$ interaction on noncommutative spaces was discussed in details in [11, 40, 47, 48], we shall not repeat it here. Using $Z\nu\bar{\nu}$ vertex from [11]², we obtain the following $Z \rightarrow \nu\bar{\nu}$ partial width

$$\begin{aligned} \Gamma(Z \rightarrow \nu\bar{\nu}) &= \Gamma_{\text{SM}}(Z \rightarrow \nu\bar{\nu}) \\ &+ \frac{\alpha}{3M_Z|\vec{E}_\theta|} \left[\kappa(1 - \kappa + \kappa \cos 2\theta_W) \sec^2 \theta_W \cos \left(\frac{M_Z^2 |\vec{E}_\theta|}{4} \right) \right. \\ &\quad \left. - 8 \csc^2 2\theta_W \right] \cdot \sin \left(\frac{M_Z^2 |\vec{E}_\theta|}{4} \right) \\ &+ \frac{\alpha M_Z}{12} \left[-2\kappa^2 + (\kappa(2\kappa - 1) + 2) \sec^2 \theta_W + 2 \csc^2 \theta_W \right], \end{aligned} \quad (11)$$

whose NC part vanishes when $\vec{E}_\theta \rightarrow 0$, i.e. for vanishing θ or space-like noncommutativity, but not

² The constant κ measures the noncommutative gauge coupling that vanishes in the commutative limit, scaled by the commutative QED coupling constant [11].

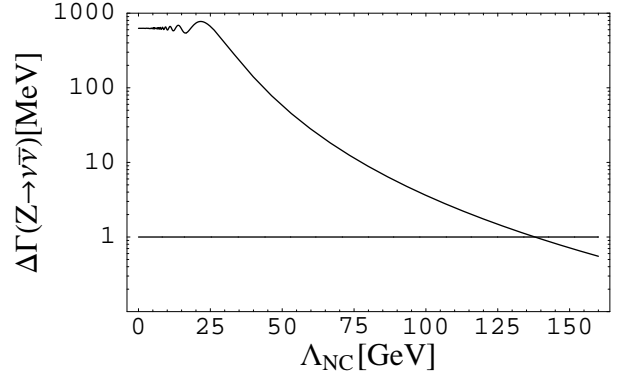


FIG. 3: $\Delta\Gamma(Z \rightarrow \nu\bar{\nu})$ decay width vs. Λ_{NC} .

light-like. A comparison of the experimental Z decay width $\Gamma_{\text{invisible}} = (499.0 \pm 1.5)$ MeV [59] with its SM theoretical counterpart, allows us to set a constraint $\Gamma(Z \rightarrow \nu\bar{\nu}) - \Gamma_{\text{SM}}(Z \rightarrow \nu\bar{\nu}) \lesssim 1$ MeV, from where a bound on the scale of noncommutativity $\Lambda_{\text{NC}} = |\vec{E}_\theta|^{-1/2} \gtrsim 140$ GeV is obtained (see Fig. 3), for the choice $\kappa = 1$.

IV. DISCUSSION AND CONCLUSION

We have shown that the tree level tri-particle decays ($Z \rightarrow \gamma\gamma, \nu\bar{\nu}$) in the covariant noncommutative quantum gauge theory based on Seiberg-Witten maps can be computed without an expansion over the noncommutative parameter θ . We obtained for the first time covariant θ -exact triple neutral gauge boson interactions within the NCSM gauge sector. We have also used results of [11] which shows explicitly that the “no-go theorem” [12] is not an issue in our SW map and enveloping algebra based approach to θ -exact NCGFT. Namely the authors in [12] failed to directly form tensor products of noncommutative fields. The proof of this failure is given in section two of [11], and explicitly stated in the last paragraph of the same section.

Focusing on Z decays into two photons and two neutrinos, we have reconsidered previous computations that were done with less sophisticated tools and derived new bounds on the scale of noncommutativity.

Inspecting Fig. 1 we see that current experimental upper limit $\text{BR}(Z \rightarrow \gamma\gamma) < 5.2 \cdot 10^{-5}$ [59] is way too weak to produce any meaningful constraint on the scale of noncommutativity. However, we can certainly expect that further analysis of the LHC experiments would significantly improve the current limit on the $Z \rightarrow \gamma\gamma$ branching ratio. Also Fig. 2 clearly shows that, for example, for BR’s as low as 10^{-14} , our noncommutative scale is $\Lambda_{\text{NC}} \gtrsim 20$ TeV, thus unobservable at the LHC energies.

Comparing to previous results, the total decay rates are modified by a factor which remains finite through-

out all energy scales. Thus, our results behave much better than the θ -expansion method when ultra high energy processes are considered. We expect that similar control on the high energy behavior can be extended to θ -exact perturbation theory involving more external fields in the near future [60]. All of our results show closed/convergent forms (see Figs. 1-3) throughout full interaction energy scales, thus facilitating further phenomenological applications. This would provide a considerably improved theoretical basis for research work in the field of noncommutative particle phenomenology.

However, at the end we do have to emphasize that a complete understanding of the θ -exact quantum loop corrections, as indicated in [47, 48], still remains in general a challenging open question [60].

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